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Surface charge of a flat superconducting slab in the Meissner state

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The electrostatic potential in the flat superconducting slab is evaluated in the framework of the Ginzburg-Landau theory extended by Bardeen to low temperatures. For magnetic fields below B_{c1}, we discuss the formation of a surface charge induced by the Bernoulli potential of the suppercurrents.

I. INTRODUCTION

A diamagnetic current in a superconductor creates an electrostatic potential known as the Bernoulli potential [1,2]. Early studies based either on the electrodynamics of the ideal charged fluid [1], on the London approximation [2] or on the local approximation of the BCS theory [3–5] provide the electrostatic potential as a function of the condensate velocity. This approach is not applicable to strongly inhomogoneous systems like Abrikosov vortices or surfaces where non-local quantum corrections are inevitable.

A simple non-local theory of the electrostatic potential in stationary superconductors has been introduced in [6]. It is based on the Ginzburg-Landau (GL) theory and neglects the so called quasiparticle screening of van Vijfeijken and Staas [7] and the thermodynamic correction predicted by Adkins and Waldram [3] and evaluated by Rickayzen [4].

A theory of Ginzburg-Landau type which covers the quasiparticle screening and the thermodynamic corrections, has been introduced in [8]. Its implementation to the Abrikosov vortex lattice is in preparation. In this paper we employ a non-local approach to discuss the formation of the surface charge. We study the simple system of a superconducting flat slab in a parallel magnetic field below B_{c1}.

The formation of a surface charge induced by the Bernoulli potential has been already studied by Jakeman and Pike [9]. Starting from the time-dependent Ginzburg-Landau theory, they have derived a Poisson equation for the electrostatic potential with screening over the Thomas-Fermi length and with a source term that includes the quasiparticle screening of van Vijfeijken and Staas [7]. Except for the Thomas-Fermi screening their approach is local, what implies that a surface charge is formed on the scale of the Thomas-Fermi screening length. Here we show that the non-local corrections to the electrostatic potential are dominant over the Thomas-Fermi screening so that the surface charge forms on the scale of the GL coherence length.

II. EXTENDED GINZBURG-LANDAU THEORY

Our treatment of the electrostatic potential starts from the free energy

$$\mathcal{F} = \int d\mathbf{r}\,(f_{\rm con} + f_{\rm kin} + f_{\rm mag}) + \mathcal{F}_{\rm Coul}. \tag{1}$$

The kinetic energy of the condensate is of the GL form [10]

$$f_{\rm kin} = \frac{1}{2m^*}\left|(-i\hbar\nabla - e^*\mathbf{A})\psi\right|^2, \tag{2}$$

where ψ is the GL wave function, $m^* = 2m$ and $e^* = 2e$ are mass and charge of a cooperon. The condensation energy follows the two-fluid model of Gorter and Casimir [11],

$$f_{\rm con} = U - \varepsilon_{\rm con}\frac{2}{n}|\psi|^2 - \frac{1}{2}\gamma T^2\sqrt{1 - \frac{2}{n}|\psi|^2} \tag{3}$$

with the condensation energy per volume given by Sommerfeld's γ as $\varepsilon_{\rm con} = \frac{1}{4}\gamma T_c^2$. The total density is the sum of the condensate and normal electrons, $n = 2|\psi|^2 + n_n$. The internal energy, U, the linear coefficient of the specific heat per volume, γ, and the critical temperature, T_c, are functions of the density n.

The magnetic free energy,

$$f_{\rm mag} = -\frac{1}{2\mu_0}\left(\mathbf{B} - \mathbf{B}_a\right)^2, \tag{4}$$

depends on the applied magnetic field B_a and the internal field $\mathbf{B} = \nabla \times \mathbf{A}$. The Coulomb energy is treated in the non-relativistic approximation,

$$\mathcal{F}_{\rm Coul} = \frac{1}{2}\int d\mathbf{r}d\mathbf{r}'\,\frac{e^2}{4\pi\epsilon}\frac{1}{|\mathbf{r} - \mathbf{r}'|}\rho(\mathbf{r})\rho(\mathbf{r}'), \tag{5}$$

where $\rho = e^*|\psi|^2 + en_n + \rho_{\rm latt}$ is the charge deviation from neutrality. We do not assume any external electric field.

Except for the Coulomb potential, this free energy has been introduced by Bardeen [12] soon after the paper of Ginzburg and Landau [10] as its simple extension to low temperatures.

In the stationary state the free energy has a minimum which can be found by the variation principle [13]. Variations with respect to $\bar{\psi}$, $\mathbf{A}$ and n_n lead to a set of extended Ginzburg-Landau equations:
the Schrödinger equation,

$$\frac{1}{2m^*}\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi + \left(-\frac{2}{n}\epsilon_{\rm con} + \frac{\gamma T^2}{2n}\frac{1}{\sqrt{1-\frac{2}{n}|\psi|^2}}\right)\psi = 0, \quad (6)$$

the Maxwell equation,

$$\nabla\times\nabla\times\mathbf{A} = \mu_0\frac{e^*}{m^*}{\rm Re}\bar{\psi}\left(-i\hbar\nabla - e^*\mathbf{A}\right)\psi. \quad (7)$$

and the screened Poisson equation,

$$e\varphi - \lambda_{TF}^2\nabla^2 e\varphi = -\frac{\partial\epsilon_{\rm con}}{\partial n}\frac{2}{n}|\psi|^2 - \frac{1}{2}T^2\frac{\partial\gamma}{\partial n}\sqrt{1-\frac{2}{n}|\psi|^2} - \frac{1}{2nm^*}\bar{\psi}\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi. \quad (8)$$

We note that variations result directly in a condition for the charge density. The electrostatic potential φ has been introduced to simplify expressions. It links to the charge density via the plain Poisson equation

$$-\varepsilon\nabla^2\varphi = e^*|\psi|^2 + en_n + \rho_{\rm latt}. \quad (9)$$

The last term on the right hand side of (8) is the nonlocal Bernoulli potential [6]. It is proportional to $|\psi|^2/n$ in the spirit of the quasiparticle screening [7]. The two other terms combine into the thermodynamic correction introduced by Rickayzen [4]. Note that the second term is the thermoelectric field of the normal metal reduced by a factor $\sqrt{1-2|\psi|^2/n}$. The screening on the Thomas-Fermi length is covered by the differential term in the left hand side of (8).

The usual GL theory has two independent variables, $\mathbf{A},\psi$ and consequently only two differential equations have to be solved. Including the electrostatic potential we have four indeterminates $\mathbf{A},\psi,\varphi,n_n$ so that the set of four coupled equations (6 - 9) should be solved. In reality the problem is not so complicated. The second GL equation (7) is independent of n_n and φ. The first GL equation (6) is independent of φ while it depends on n_n exclusively via $n = n_n + 2|\psi|^2$. Since n is very large in metals and charge perturbations are rather small, one can neglect deviations from the charge neutrality in the first GL equation. In this approximation, equations (6,7) are solved separately. Once $\mathbf{A}$ and ψ are known, equation (8) provides us with the electrostatic potential and the charge is found from the Poisson equation (9).

III. FLAT SLAB IN MAGNETIC FIELD

Let us demonstrate the behavior of the electrostatic potential in a superconducting slab in parallel magnetic field. We assume a superconductor of type II in a magnetic field below B_{c1}, i.e., in the Meissner state with the magnetic field penetrating from the surface.

We take the direction x perpendicular to the slab limited to the interval $(-d,d)$. The magnetic field we choose in the direction z, $\mathbf{B}\equiv(0,0,B)$, and the vector potential points in the direction y, $\mathbf{A}=(0,A,0)$. All functions depend exclusively on the coordinate x and the wave function is real.

We rescale all functions into their dimensionless counterparts, $x=\tilde{x}\lambda_0$, $A=\tilde{A}\Phi_0/(2\pi\lambda_0)$, $\psi=\tilde{\psi}\sqrt{n/2}$, $e\varphi=\tilde{\varphi}\hbar^2/(4m\lambda_0^2)$, where the London penetration depth at zero temperature is given by $\lambda_0^2=m/(e^2n\mu_0)$. Note that our scale does not change with the temperature. In the expressions below we skip tildes denoting new functions. Equations (6,7) now read

$$\frac{\partial^2 A}{\partial x^2} - A\psi^2 = 0, \quad (10)$$

$$\frac{\partial^2\psi}{\partial x^2} - A^2\psi + S\left(1-\frac{t^2}{\sqrt{1-\psi^2}}\right)\psi = 0, \quad (11)$$

with $S=\kappa^2(1-t^2)^2(1+t^2)$. As usual in the GL theory, the only material parameter relevant after rescaling is the GL parameter κ defined at T_c. In the Bardeen extension the rescaled equations also depend on the reduced temperature $t=T/T_c$.

As the boundary condition we use that the vector potential is anti-symmetric, $A(-x)=-A(x)$, with the value of the derivative at the surface given by the applied magnetic field B_a. The wave function has a zero derivative at the surface.

The third GL equation (8) in the dimensionless units,

$$\varphi - \tau^2\frac{\partial^2\varphi}{\partial x^2} = C_1\psi^2 + 2C_2t^2\sqrt{1-\psi^2} - \left(1-\frac{t^2}{\sqrt{1-\psi^2}}\right)\psi^2, \quad (12)$$

depends on two material parameters,

$$C_1 = \frac{\partial\ln\varepsilon_{\rm con}}{\partial\ln n}, \quad (13)$$

$$C_2 = \frac{\partial\ln\gamma}{\partial\ln n}. \quad (14)$$

For Niobium these parameters can be derived from the McMillan formula [14] and chemical trends [15] giving values $C_1=1.9$ and $C_2=0.42$.

The reduced Thomas-Fermi screening length,

$$\tau = \frac{\lambda_{TF}}{\lambda_0}, \tag{15}$$

is very small. For Niobium $\lambda_{TF} = 0.7$ Å and $\lambda_0 = 390$ Å. The Thomas-Fermi screening thus enter equation (12) with factor $\tau^2 = 3 \cdot 10^{-6}$. With accuracy of the order of 10^{-5}, the solution of (12) is $\varphi = \varphi_{\rm in} + \varphi_{\rm free}$, with the induced term,

$$\varphi_{\rm in} = C_1\psi^2 + 2C_2 t^2\sqrt{1-\psi^2} - \left(1 - \frac{t^2}{\sqrt{1-\psi^2}}\right)\psi^2, \tag{16}$$

and the free solution

$$\varphi_{\rm free} = \varphi_0 \cosh(x/\tau). \tag{17}$$

The free solution decreases near the surface on the scale of the Thomas-Fermi screening length $\tau = 1.8 \cdot 10^{-3}$.

The amplitude of the free solution has to be selected so that the slab remains charge neutral. In other words, the free solution supplies the surface charge which is localized on the Thomas-Fermi screening length. The neutrality of the slab is equivalent to the condition that the electric field vanishes at the surface. The amplitude of the free solution is thus given by the condition $\partial\varphi/\partial x = 0$ on the surface.

Following [16] it is possible to show that the free solution is zero. The induced potential $\varphi_{\rm in}$ has zero derivative on the surface as it follows from (16) and the GL boundary condition, $\partial\psi/\partial x = 0$. Accordingly, in the non-local approach one has $\varphi_0 = 0$.

IV. LOCAL VERSUS NON-LOCAL PICTURE

The local approximation corresponds to a neglect of the gradients in the GL equation (11). The density of the condensate, ψ^2, is thus found from a simple local condition,

$$1 - \frac{t^2}{\sqrt{1-\psi^2}} = \frac{A^2}{S}, \tag{18}$$

From (18) one obtains ψ^2 as a function of the vector potential A. This function is first used in (10) to solve for A and finally substituted into (12) to find the electrostatic potential φ.

The wave function ψ found within the local approximation (18) does not satisfy the GL boundary condition $\partial\psi/\partial x = 0$. Accordingly, the derivative of the induced potential is non-zero, $\varphi_{\rm in} \neq 0$, so that the free solution has to be supplemented to maintain the charge neutrality.

Now we compare the full solution of (10 - 12) with the local approximation. In our treatment we assume Niobium with Oxygen impurities of concentration giving $\kappa = 1.5$. The width of the slab we take $6\lambda_0$ so that the magnetic field is screened from the bulk at low temperatures but penetrates the whole slab close to T_c.

At low temperatures one finds that the magnetic field suppresses the condensate only in a negligible fraction. Since the condensate density is nearly constant, it follows from (10) that the magnetic field differs only negligibly from the London approximation, $B = B_0\cosh(x/\lambda)$, with $B_0 = B_a/\cosh(d/\lambda)$ and $\lambda \approx 1$. Of course, since corrections beyond the London approximation are negligible, the local and non-local approaches yield very similar magnetic fields.

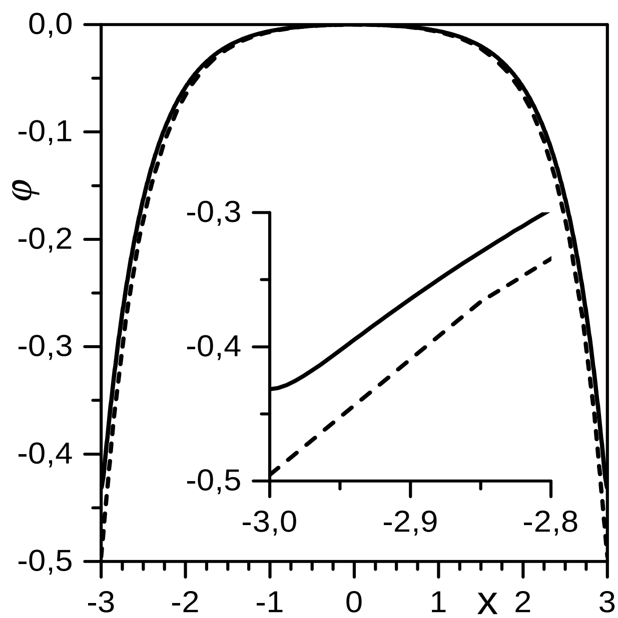

FIG. 1. The electrostatic potential at $t = 0.1$ and $B \approx B_{c1}$. The local approximation (dashed line) differs from the full non-local solution (full line) mainly close to the surface shown in the insert.

In figure 1 we show the electrostatic potential. The local approximation agrees with the non-local approach very well except for the close vicinity of the surface. This region is affected by the GL boundary condition. The presented result is for a magnetic field close to B_{c1}.

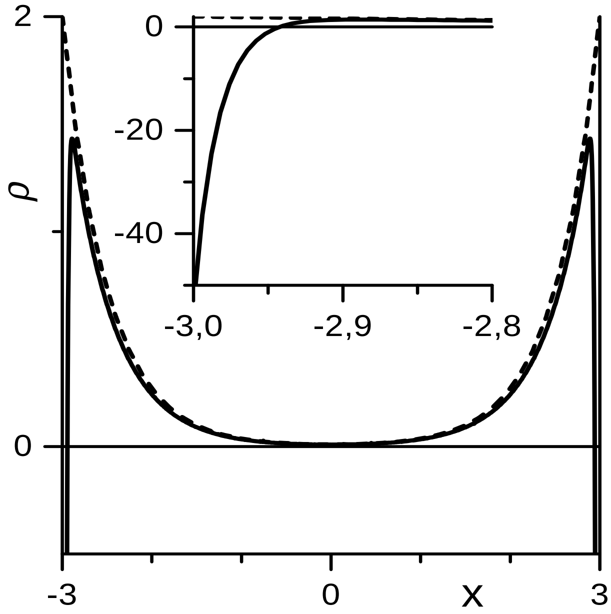

FIG. 2. The charge distribution corresponding to the electrostatic potential from Fig. 1.

The formation of the surface charge is shown in Fig. 2. The depleted charge appears in the surface layer of characteristic width which is small compared to the London penetration depth or the GL coherence length, however still very large on the scale of the Thomas-Fermi screening.

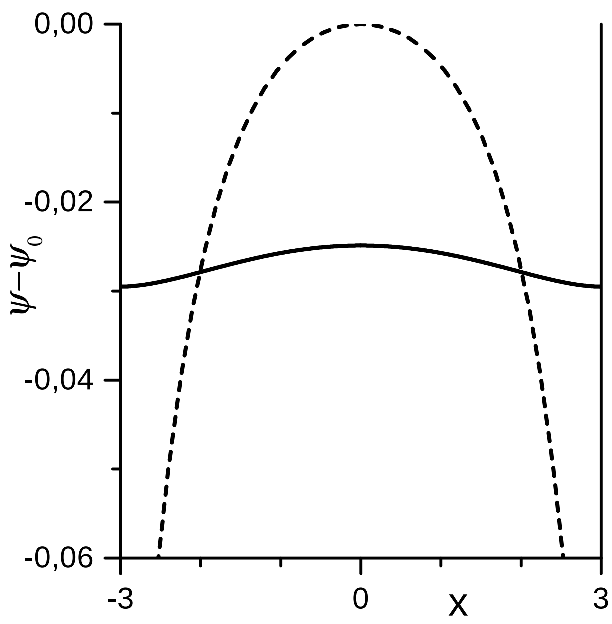

FIG. 3. Deviation of the wave function ψ from its equilibrium value, $\psi_0 = \sqrt{1-t^4} \approx 0.6$ for $t = 0.9$. Already for a weak field, $B = 0.1\, B_{c1}$, the local approximation (dashed line) is very different from the full non-local solution (full line).

For temperatures close to T_c the local approximation becomes unreliable if one looks for the electrostatic potential or the related charge distribution. This contrasts with the magnetic properties. For temperature $t = 0.9$ numerical results confirm that for a weak applied field, $B = 0.1\, B_{c1}$, the magnetic field is quite well described by the London approximation, $B = B_0 \cosh(x/\lambda)$, with $B_0 = B_a/\cosh(d/\lambda)$ and $\lambda = 1.7$. It again follows from a small effect of the magnetic field on the wave function which is nearly constant across the slab and reduced by 5% compared to the bulk value $\psi_0 = \sqrt{1-t^4} = 0.6$, see Fig. 3. One can see that the local approximation for the deviation of the wave function is rather bad in this case. Indeed, the coherence length is comparable to the width of the slab so that the GL boundary condition is essential in the whole volume.

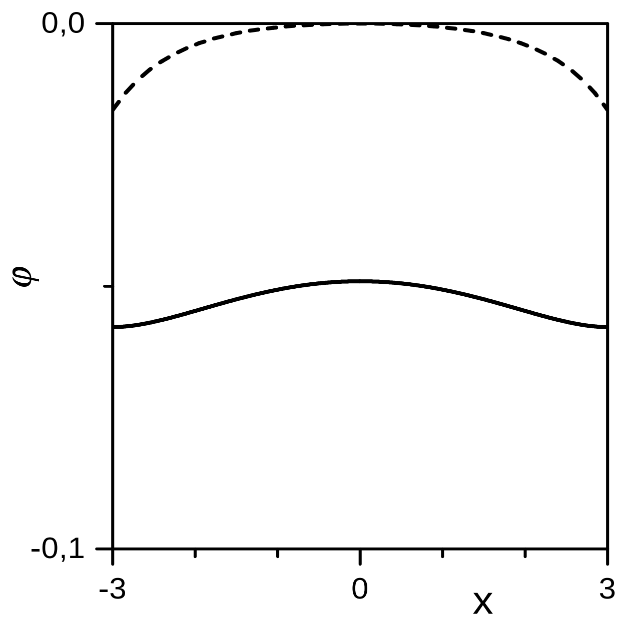

FIG. 4. The electrostatic potential $\varphi = \varphi_{\rm in} + \varphi_{\rm free}$ for $t = 0.9$ and $B = 0.1\, B_{c1}$. The constant term is chosen so that both potentials (16) and (17) would reach zero in the bulk of a thick sample.

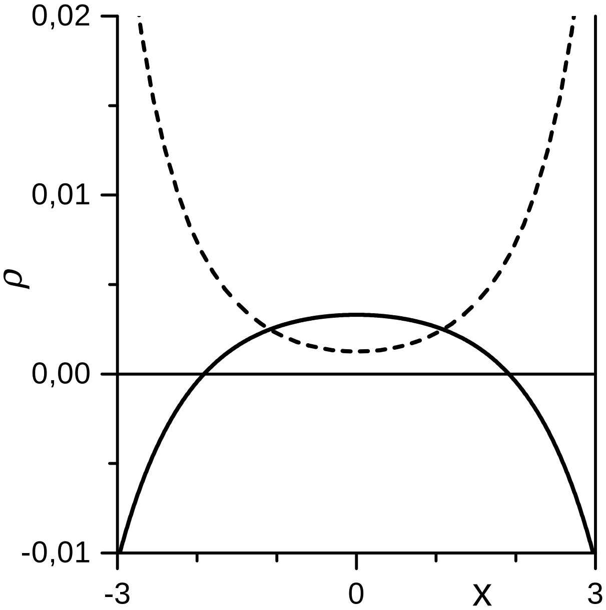

FIG. 5. The charge distribution corresponding to the electrostatic potential from Fig. 4.

The local and non-local values of the electrostatic potential shown in Fig. 4 differ appreciably. First, the non-local potential is shifted down compared to the local approximation. We note that in both approximations the initial is identified with the bulk of a thick sample. The constant shift, however, is not important for the distribution of the charge shown in Fig. 5. One can see that in the local approximation the charge is positive everywhere. The charge neutrality is maintained by an invisibly thin depleted layer on the scale $\tau \approx 10^{-3}$. In the non-local approach, the layer of the depleted charge extends over the region comparable with the GL coherence length.

In conclusion, the formation of the surface charge due to the electrostatic potential caused by diamagnetic currents has been treated within the local and non-local approaches. While the local approach requires to include the surface on the scale of the Thomas-Fermi screening length, the non-local approach predicts that the surface charge extends over a layer of width comparable with the coherence length.

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